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LAMINAR, TRANSITION AND TURBULENT NATURAL CONVECTION ADJACENT TO INCLINED AND VERTICAL SURFACES

J. R. LLOYD*, E. M. SPARROW and E. R. G. ECKERT
Department of Mechanical Engineering, University of Minnesota, Minneapolis, Minnesota, U.S.A.

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Abstract—Experiments on natural convection mass transfer adjacent to vertical and upward-facing inclined plates were performed by employing an electrochemical technique. Local measurements were made, yielding both streamwise and spanwise mass transfer distributions. Time-dependent mass transfer rates were measured in the transition and turbulent regimes.

The measured local laminar mass transfer coefficients agreed very well with analytical predictions, both for vertical and inclined surfaces. The Rayleigh number marking the onset of laminar-turbulent transition varied markedly with the angle of inclination, decreasing with increasing departures from the vertical. In the transition regime, for surfaces inclined at angles greater than 15 degrees to the vertical, significant spanwise variations in both the instantaneous and time-averaged mass transfer rates were in evidence. The timewise fluctuations of the turbulent mass transfer rates were of substantially greater amplitude and period for inclined surfaces than for the vertical plate. The time-averaged, local turbulent mass transfer coefficients were correlated with the one-third power of the Rayleigh number and, in addition, exhibited an appreciable dependence on the inclination angle.

NOMENCLATURE

C, angle-dependent coefficient, equation (15);
c, concentration of transferred ion at the test surface;
c_∞, concentration of transferred ion in the fluid bulk;
D, diffusion coefficient;
F, Faraday constant;
g, acceleration of gravity;
$Gr_{\theta,x}$, local Grashof number, $\rho g \cos\theta (\rho_\infty - \rho_w)x^3/\mu^2$;
i, current density;
K_x, local mass transfer coefficient, equation (8);
$\dot{N}$, local rate of mass transfer per unit area;
$\dot{N}_c, \dot{N}_d, \dot{N}_m$, mass transfer rates due to convection, diffusion and migration;
n, valence of transferred ions;
$Ra_{\theta,x}$, local Rayleigh number, $Gr_{\theta,x}Sc$;
Sc, Schmidt number, $\mu/\rho D$;
Sh_x, local Sherwood number, $K_x x/D$;
t, transference number;
U, mobility;
v, transverse velocity at the test surface;
x, streamwise coordinate;
y, transverse coordinate;
θ, angle of inclination, Fig. 1;
μ, viscosity;
ρ, density;
ρ_w, density at the test surface;

* Present address: Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, Indiana, U.S.A.

ρ_∞, density in the fluid bulk;
ϕ, voltage.

INTRODUCTION

NATURAL convection adjacent to vertical and horizontal surfaces has been extensively investigated, both analytically and experimentally. On the other hand, there has been relatively little study of natural convection adjacent to inclined surfaces. The present paper describes a broad-ranging experimental investigation of natural convection adjacent to inclined, upward-facing plane surfaces, including the vertical plate. The experiments encompassed the laminar, transition, and turbulent flow regimes.

An electrochemical technique was employed which facilitates highly accurate, local measurements of natural convection mass transfer. Furthermore, since the mass flux sensors are free of inertia, they were able to detect the time variations which occur in the transition and turbulent regimes. The electrolyte was an aqueous solution of cupric sulphate and sulphuric acid.

The boundary condition for the experiments was uniform concentration at the plate surface, which is the counterpart of uniform surface temperature in the corresponding heat transfer problem. The Schmidt numbers were on the order of 2000, so that the results found here should also be relevant to high Prandtl number, natural convection heat transfer. In this connection, it may be noted that the present mass transfer experiments were performed under nearly constant property conditions, whereas large property variations generally intrude in high Prandtl number heat transfer experiments.

Local measurements were made in the laminar regime and compared with the vertical plate prediction and with its generalization for inclined surfaces. The conditions marking the breakdown of the laminar regime were detected both from data point deviations and from the observed time variation of the measured mass transfer rates. With respect to laminar breakdown, it has been shown [1] that the nature of the flow instability is different depending on the angle of inclination of the surface. In particular, for inclination angles relative to the vertical greater than about 15 deg, the onset of transition was found to be characterized by the presence of longitudinal vortices which are distributed in a fairly regular pattern across the horizontal span of the plate surface. The presence of such vortices would be expected to cause spanwise variations of the mass transfer (or heat transfer). The measurement of spanwise mass transfer variations in the transition regime was one of the major objectives of this research. Such measurements were made instantaneously to accommodate a timewise shifting of the vortex lines, with both instantaneous and time-averaged results being presented. In the turbulent regime, where spanwise uniformity is restored, instantaneous mass transfer rates were measured and subsequently time averaged.

Background literature relevant to the present investigation will now be briefly discussed. Heat transfer measurements on inclined, upward-facing surfaces appear to have been reported only by Rich [2], Kierkus [3] and Vliet [4]. The experiments of Rich and Kierkus were made in air and were concerned mainly with the laminar range. Rich's data exhibit a substantial amount of scatter and have been questioned by other authors, while Kierkus' work was confined to a single angle of inclination and is, therefore, insufficient to permit definite conclusions as to the effect of inclination. Vliet employed water as his primary working fluid* in conjunction with a constant heat flux surface oriented at several angles of inclination; the results cover the laminar, transition, and turbulent regimes. Natural convection mass transfer on inclined surfaces was discussed in a qualitative manner by Wagner [5], but actual data were not presented.

Results for turbulent natural convection adjacent to isothermal or isoconcentration inclined surfaces are essentially non-existent in

* Two data runs were made with air.

the literature, and laminar results are only sparsely available. Local transfer coefficients, unaffected by large fluid property variations, are also lacking for high Prandtl or Schmidt number natural convection adjacent to vertical surfaces. Spanwise variations of natural convection, transition-regime heat or mass transfer have not been heretofore explored, nor has the time-dependence of the mass transfer in the transition and turbulent regimes.* No local mass transfer measurements appear to have been reported for either vertical or inclined surfaces, regardless of the flow regime.

EXPERIMENTAL APPARATUS AND MEASUREMENT TECHNIQUE

Inasmuch as the local mass transfer measurements performed during the course of this research were of a type not previously reported in the literature, a substantial amount of apparatus development was necessary. An account of the developmental work is given elsewhere [7], as are the fine details of the apparatus and instrumentation. The present description will be limited to the more important features of the apparatus and its use.

The right-hand diagram of Fig. 1 is an overall schematic of the experimental set-up which, to simplify the discussion, shows a preliminary test surface and its elementary electric circuit rather than the test surface and circuitry used for local data acquisition. The latter will be described shortly. As illustrated in this diagram, the test chamber was a rectangular tank (30-gallon capacity), made of polyethylene and having dimensions of 61 × 45·6 × 45·6 cm (length × width × depth). The tank contained an electrolyte consisting of aqueous solutions of reagent grade cupric sulphate (0·03–0·08 M) and sulphuric acid (1·5 M). The copper ions of the cupric sulphate served as the transferred species, whereas the sulphuric acid was the supporting electrolyte.

The cathode served as the test surface, whereas the anode was a 40 cm square copper sheet, 0·08 cm thick. Preliminary experiments indicated that the mass transfer at the cathode was insensitive to the placement of the anode. The final positioning of the cathode and anode is shown in the diagram. Electric power was supplied by a 12V automotive storage battery, and the voltage between the fluid bulk and the cathode, measured with a reference electrode consisting of a fine copper wire housed in an open-ended teflon tube, was controlled by a rheostat. The current flow from the anode to the cathode, which, as will be demonstrated later, is proportional to the rate of mass transfer at the

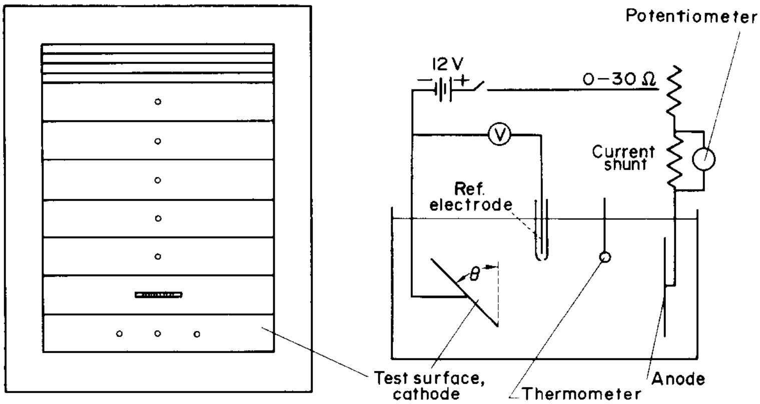

FIG. 1. Schematic diagrams of the experimental apparatus.

* Timewise variations of the wall temperature of an electrically heated vertical cylinder in water were measured by Fujii and co-workers [6], but the thermal inertia of the wall might well have affected the results.

cathode, was measured in terms of the voltage drop across a calibrated precision resistor (hereafter designated as a current shunt).

Attention may now be turned to the test surface and corresponding instrumentation employed during the investigational phase of this work. A front-face schematic diagram of the test surface is given in the left-hand portion of Fig. 1, where a segmented construction is clearly in evidence. The locations of the eleven individual segments could be varied at will, and the arrangement shown in the diagram is illustrative of the many that were used. All segments were of nickel 200, 15·2 cm in spanwise width and 0·635 cm thick. Seven of the segments, those instrumented for mass transfer measurements, were 2·54 cm high, whereas the four remaining segments, which served to vary the streamwise locations of the mass transfer probes, were 0·635 cm in height. Each segment was fitted on its rear surface with an electrical binding post by means of which electrical connection was made to the power supply. Under operating conditions, all segments of the test surface were at the same uniform voltage, and the plate temperature was identical to that of the fluid environment.

The active test surface had overall dimensions of 15·2 × 20·3 cm (spanwise width × height), and, as depicted in the diagram, it is set into a 2·54 cm wide teflon frame. In turn, the teflon frame was supported by a stand that could be positioned so as to orient the test surface at any one of several angles of inclination with respect to the vertical. The stand rested on the test chamber floor, which was carefully aligned to the horizontal.

As may be seen in the diagram, six of the segments were instrumented with small circular electrodes which served as mass transfer probes. Each probe was approximately 0·343 cm dia. and was electrically insulated from its host bar by a 0·0127 cm annular gap filled with epoxy. The probes were fabricated from nickel 200. The seventh instrumented segment housed an ensemble of 55 probes designed for the measurement of spanwise mass transfer distributions. Each probe, fabricated from nickel 200 shim stock, had a spanwise extent of 0·0254 cm, and insulation between probes was provided by 0·0051 cm polyethylene sheeting. The ensemble of probes encompassed a spanwise length of 1·68 cm and a streamwise length of 0·318 cm. Electrical isolation of the probe ensemble from the host bar was accomplished by Glyptal insulating paint and by epoxy. In actuality the probe ensemble unit was a complex piece of instrumentation, requiring precise fabrication and assembly. A detailed description of this instrument, as well as more information about the test surface in general, is available elsewhere [7].

Each one of the probes was incorporated into an electric circuit similar to that already discussed in connection with the right-hand diagram of Fig. 1, with a separate current shunt being used for each probe. The current shunts for the eight circular probes were 15 Ω calibrated precision resistors, whereas those for the ensemble of 55 probes were one per cent 174 Ω precision resistors. The current shunt readings for laminar flow conditions were measured with a Leeds and Northrop K-3 potentiometer. Current flow data for all other operating conditions were taken with a 24-channel Dymec digital voltmeter, which could be programmed to read the probe outputs at preselected time intervals. A detailed circuit diagram is given in [7].

One of the conditions essential to the acquisition of accurate mass transfer data was that spurious current flows be eliminated. In order to fulfill this requirement, all exposed metallic surfaces, other than the test surface itself, were painstakingly insulated, and the regions of contact between the probes and the host bars were examined before and after each data run (see [7] for details). Another important aspect of the experimental procedure was the renewing and refinishing of the surfaces of the test section after each data run, such resurfacing being required owing to the deposition of copper ions by the electrochemical reaction.

All data were taken for the limiting current operating condition, which is characterized by a zero concentration of the transferred ion (i.e. copper) at the cathode, thereby obviating the need to measure surface concentrations. Limiting current conditions were identified by varying the voltage between the fluid bulk and the test surface and measuring the corresponding variations in shunt current for one or several probes. For sufficiently high copper ion concentrations in the fluid bulk, a graph of current versus voltage displayed a more or less horizontal plateau in the range between 0·35 and 0·55 V. Such a plateau is indicative of the limiting current condition. A substantial amount of voltage–current data were taken in order to define the range of bulk concentrations that permitted limiting current conditions to be attained.

The composition of the bulk solution was obtained by sampling. The concentrations of the copper ions and sulphuric acid were respectively measured by spectrometric and titration methods, respectively described in [8] and [9]. No change in bulk fluid composition was detected during the course of a data run. Vertical gradients were also monitored and found to be very small.

DATA ANALYSIS

Consideration is now given to the relationship between the measured electric current flow and the mass transfer rate at the surface of a probe. The mass transfer coefficient and dimensionless moduli used to represent the results will also be discussed.

The principle mechanisms for the transfer of copper ions to an element of cathode surface (e.g. a probe) are diffusion, convection, and migration. If $\dot{N}$ represents the local mass rate of copper ion transfer per unit area due to all mechanisms, and $\dot{N}_d$, $\dot{N}_c$ and $\dot{N}_m$ are the corresponding fluxes due to diffusion, convection, and migration, then

$$\dot{N} = \dot{N}_d + \dot{N}_c + \dot{N}_m. \tag{1}$$

In turn, the component fluxes are representable as [10]

$$\dot{N}_d = -D\partial c/\partial y,\ \dot{N}_c = vc,\ \dot{N}_m = Uc\partial\phi/\partial y \tag{2}$$

in which D and U respectively denote the diffusion coefficient and the mobility; in addition, c, $\partial c/\partial y$, $\partial\phi/\partial y$ and v are, respectively, the copper ion concentration, the concentration gradient, the voltage gradient, and the transverse velocity, all taken at the surface.

Equation (1) is to be applied to measurements made under limiting current conditions, so that c is vanishingly small. Correspondingly, major simplifications can be made in equation (1). Consider first the convection term $\dot{N}_c = vc$. To estimate the magnitude of the transverse velocity v, the electrolyte solution may be assumed to be a binary mixture with the copper ions as one component and the remaining constituents as the other. Then, since only the copper ions are absorbed by the wall, it can be shown (e.g. [11]) that $v = -[D/(1-c/\rho)]\partial(c/\rho)/\partial y$ With this, one can group the convection and diffusion terms by taking their sum, and upon specializing to limiting current conditions, there is obtained

$$\dot{N}_c + \dot{N}_d = -\left(\frac{c/\rho}{1-c/\rho} + 1\right) D\frac{\partial c}{\partial y} \cong -D\frac{\partial c}{\partial y}. \tag{3}$$

Next, considering the migration term $Uc\partial\phi/\partial y$, it may be noted that although c is very small under limiting current conditions, the gradient $\partial\phi/\partial y$ is very large owing to the presence of the supporting electrolyte (sulphuric acid). Thus, the product $c\partial\phi/\partial y$ may be finite. A representation of the migration term for limiting current conditions is given in [10] as

$$\dot{N}_m = ti/nF \tag{4}$$

where t is the transference number for the copper ions* at the average copper ion and sulphuric acid concentrations, i is the current density, n is the valence of the copper ions, and F is the

* t is a measure of the portion of the current carried by the copper ions.

Faraday number. In the presence of a sufficiently strong supporting electrolyte, essentially all of the current passing through the solution is carried by the supporting electrolyte and almost none by the copper ions; consequently, $t \ll 1$.

The rate of ion transfer by all mechanisms is given in [10] as

$$\dot{N} = i/nF. \tag{5}$$

Since n and F are known constants, it follows that the mass transfer rate $\dot{N}$ is directly proportional to the current density. Furthermore, from equations (4) and (5)

$$\dot{N} - \dot{N}_m = (1 - t)\, i/nF = (1 - t)\, \dot{N}. \tag{6}$$

Then, upon bringing together equations (1), (3) and (6), and then noting that $\dot{N}_d = -D\, \partial c/\partial y$, there results

$$\dot{N} = (-D\, \partial c/\partial y)/(1 - t) = \dot{N}_d/(1 - t). \tag{7}$$

Inasmuch as $(1 - t) \approx 1$, it may be omitted. Equation (7) indicates that the surface mass transfer is virtually equal to the diffusive flux, thereby fulfilling a necessary condition for the validity of the analogy between heat and mass transfer.

The local mass transfer coefficient K_x may now be defined as

$$K_x = \dot{N}_d/(c_\infty - c) \tag{8}$$

where c_∞ and c respectively denote the concentrations of the transferred ion in the fluid bulk and at the surface. $\dot{N}_d$ can be conveniently expressed via equations (7) and (5), and c is essentially zero for the limiting current condition, so that

$$K_x = i(1 - t)/nFc_\infty \tag{9}$$

from which it is seen that numerical values of K_x are determined directly from the measured current densities and bulk copper ion concentrations.*

The Sherwood number serves as a dimensionless representation of the mass transfer coefficient and plays a role analogous to the Nusselt number for heat transfer. The local Sherwood number Sh_x may be defined as

$$Sh_x = K_x x/D. \tag{10}$$

Another relevant dimensionless group is the Rayleigh number, which, for mass transfer problems, is equal to the product of the Grashof and Schmidt numbers. In constructing the Grashof number, the component of the body force parallel to the test surface will be used, that is, $g \cos \theta$, and the designation Gr_θ is adopted to indicate the presence of $\cos \theta$. Furthermore, the density difference between the bulk fluid and the surface, $\rho_\infty - \rho_w$, will appear directly in the Grashof number (in contrast to heat transfer problems, where it is traditional to use an approximate representation for the density difference). Thus, the local Grashof number $Gr_{\theta,x}$ is

$$Gr_{\theta,x} = \rho g \cos \theta\, (\rho_\infty - \rho_w)\, x^3/\mu^2. \tag{11}$$

In addition, the Schmidt number is defined as

$$Sc = \mu/\rho D \tag{12}$$

and, finally, the Rayleigh number $Ra_{\theta,x}$ is given by

$$Ra_{\theta,x} = Gr_{\theta,x}\, Sc. \tag{13}$$

As was mentioned earlier, the bulk-to-wall fluid property variations were very small, typically on the order of one per cent. The quantities D, ρ and μ appearing in equations (10)–(12) were evaluated at the average of the concentrations at the wall and in the fluid bulk. Detailed information about the evaluation of the fluid properties is given in [7].

RESULTS AND DISCUSSION

Before proceeding with the presentation of the main results, it is relevant to discuss briefly certain preliminary results which provide interesting insights into the quality of the experimental apparatus. The experiments in question were performed for laminar flow conditions and with the plate surface vertical. However,

* The transference number t, although very small ($\sim 0{\cdot}002$), was retained and evaluated as described in [7].

instead of the orientation shown in the left-hand diagram of Fig. 1, the plate assembly was turned through 90 deg so that the main line of six circular probes was situated along a horizontal, each probe being at the same distance from the leading edge. Furthermore, various placements of the anode were employed and, in some tests, one or more of the nickel bars were replaced by non-conducting plexiglas spacers, the objective being to determine vertical plate, the local Nusselt or Sherwood numbers are proportional to the fourth root of the Rayleigh number. The proportionality "constant" is a slowly varying function of the Prandtl or Schmidt numbers in the range of high Pr or Sc. For $Sc = 2000$, which is appropriate to the present experiments, interpolation in a compilation [12] of available analytical solutions for isothermal or iso-concentration vertical plates leads to

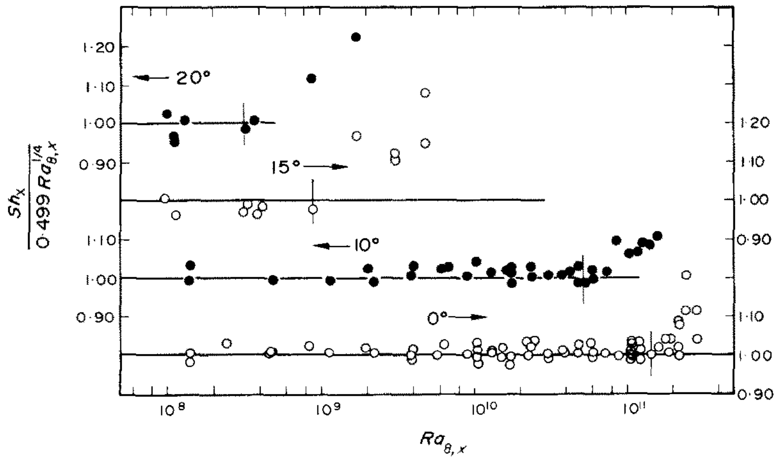

FIG. 2. Local laminar mass transfer results.

the influence of electrode orientation and of edge effects. Probes were situated as close as 1·7 cm from the side edges. The measured mass transfer results showed no systematic variation with spanwise location and with proximity to the side edges, regardless of the position of the anode or of the absence or presence of the non-conducting segments. Furthermore, the average deviation of the data from laminar theory was only 1·6 per cent. These findings lend confidence to the quality of the experimental apparatus.

The main results will now be presented. For all of these experiments, the orientation of the test surface was that pictured in the left-hand diagram of Fig. 1.

Laminar regime and onset of transition

It is well known that for laminar, natural convection heat or mass transfer adjacent to a

$$Sh_x = 0{\cdot}499\, Ra_x^{\frac{1}{4}}. \tag{14}$$

To accommodate inclined plates, equation (14) will be generalized by replacing g with $g \cos\theta$, so that

$$Sh_x = 0{\cdot}499\, Ra_{\theta,x}^{\frac{1}{4}}. \tag{14a}$$

Although usual practice suggests a presentation of results where Sh_x is plotted vs. $Ra_{\theta,x}$ on logarithmic coordinates, an alternate representation is used here which is much more sensitive to small departures of data from the $Sh_x \sim Ra_{\theta,x}^{\frac{1}{4}}$ relationship. Such increased sensitivity is attained when $Sh_x/0{\cdot}499 Ra_{\theta,x}^{\frac{1}{4}}$ is plotted against $Ra_{\theta,x}$. For laminar flow conditions, data represented in this way should fall along a horizontal line. When a suitable linear scale is used for the ordinate, data point displacements of one per cent are clearly visible.

Figure 2 shows local mass transfer data plotted in the manner just described. Results are presented for four angles of inclination, 0, 10, 15 and 20 deg. The left- and right-hand ordinates are used alternately for the successive angles, as indicated by the arrows. The horizontal lines corresponding to ordinate values of unity indicate the prediction of laminar theory, equation (14), and its generalization, equation (14a). Results for inclination angles greater than 20 deg are not shown in the figure, since, for larger angles, the flow at the lowermost measurement station was not laminar.

Attention is first turned to the results for the vertical plate. It is seen that, aside from the highest Rayleigh numbers (where the onset of transition occurs), there is remarkable agreement between the local Sherwood numbers of experiment and those of analysis. Indeed, except for a single data point, the experimental results are within three per cent of the analytical prediction in the entire laminar range. It is believed that this level of agreement provides the strongest available confirmation of the predictions of laminar boundary layer theory.

The agreement between the data for 10 deg inclination and the generalized analytical prediction, equation (14a), is as good as that for the vertical plate. For the 15 and 20 deg inclinations, there is agreement to within four per cent. It appears, therefore, that within the limited range of angles dealt with in Fig. 2, the Sherwood number results are well represented by equation (14a). Previous work by Vliet [4] on natural convection adjacent to a uniformly heated inclined plate in water has also indicated that the use of $g \cos \theta$ in place of g is an adequate generalization of the vertical plate predictions in the laminar regime.

The results of Fig. 2 can be employed to deduce information about the onset of laminar-turbulent transition. One indication of the onset of transition is the systematic upward departure of the data points from the horizontal line which corresponds to laminar flow conditions. A second indication may be obtained from the nature of the current flow measurements. At the smaller Rayleigh numbers, the readings were very steady. However, at Rayleigh numbers just below those at which the data lifted off the laminar line, small fluctuations in the readings were observed. The Rayleigh number at which these fluctuations were first noted is indicated in Fig. 2 by a short vertical line for each angle of inclination. The difference between the Rayleigh numbers at which fluctuations appeared and at which data point deviations occurred is not large. However, it is clear that data point deviations are a somewhat tardy indicator of laminar breakdown.

For the vertical plate, the onset of transition is seen to occur at a Rayleigh number of approximately 2×10^{11}. Prior investigations [10, 13, 14] of electrochemical mass transfer for vertical plates have reported instability Rayleigh numbers in the range $4\text{–}5 \times 10^{11}$. Since the prior results were deduced from overall mass transfer data and from the traditional logarithmic plot, they are, presumably, somewhat less accurate than that found here; nevertheless, there is generally good agreement. In contrast, for thermal natural convection adjacent to a vertical plate in air or in water, a recent compilation [1] of available information suggests an instability Rayleigh number of approximately 8×10^8.

Two tentative explanations may be offered to illuminate the aforementioned large difference between the instability Rayleigh numbers for thermally induced natural convection in air or water and for concentration induced natural convection at high Schmidt numbers. First, as already suggested in [14], the Rayleigh number may not be a unique criterion for instability; rather, there may be a separate dependence on the Prandtl or Schmidt numbers. The results of recent heat transfer experiments by Fujii and co-workers [6] lend some support to this hypothesis. These experiments, performed with a vertical cylinder situated in various liquids, covered a Prandtl number range from 2 to 2600. Although there is some uncertainty owing to large variable property effects, a trend can be discerned showing higher instability Rayleigh numbers at higher Prandtl numbers.

A second explanation of the Rayleigh number difference relates to the natural disturbances that may be present in the fluid environment. It is quite likely that the environmental disturbances in the mass transfer studies were smaller than those of the heat transfer studies, and the frequencies were probably different. There is no direct evidence as to the role that might have been played by these factors.

The instability results for inclined surfaces will now be discussed. From Fig. 2, it is seen that the instability Rayleigh number decreases as the angle of inclination becomes greater. A similar trend, but with a more gradual decrease, is in evidence among the available results for thermally induced natural convection [1]. Figure 2 shows a very sharp change in the instability Rayleigh number between 10 and 15 deg. This might well be due to a change in the nature of the instability which occurs in this range [1], that is, from plane wave disturbances to longitudinal vortices.

Transition and turbulent regimes

The local mass transfer rates vary with time in the transition and turbulent regimes. Since the mass transfer probes are free of inertia, they are able to follow the timewise variations. The time history of the instantaneous mass transfer rate at any one of the circular probes was obtained by programming the digital voltmeter to repetitively read the shunt voltage at preselected intervals of time. Portions of representative time histories are shown in Fig. 3 for inclination angles between 0 and 45 deg, with the shunt signal in mV on the ordinate and the time in seconds on the abscissa. Different ordinate scales are used for various inclination angles. The Rayleigh numbers for the time histories are noted on the figure and are indicative of the turbulent regime.

From the figure, it is seen that both the frequency and amplitude of the local mass transfer fluctuations experienced by the vertical plate were markedly different from those for inclined plates. For the latter, the fluctuation period is on the order of 30 s whereas the

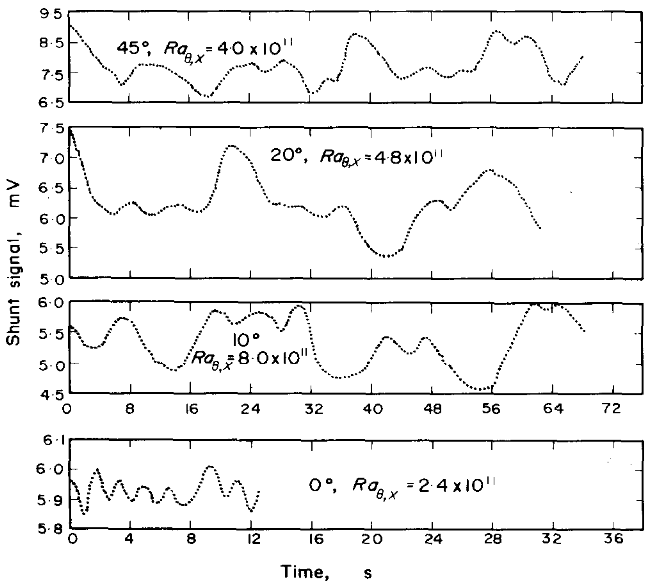

FIG. 3. Representative timewise variations of the local mass transfer in the turbulent regime.

typical fluctuation period for the former is about two seconds. There is also an order of magnitude difference in the amplitudes. For the inclined plates, peak-to-valley variations which are about 30 per cent of the mean signal are in evidence.

In connection with the aforementioned fluctuation frequencies, it is relevant to mention the boundary layer temperature measurements of Lock and Trotter [15] for turbulent natural convection adjacent to a vertical plate in water. These experiments showed that the frequency of the temperature fluctuations was substantially smaller in the neighborhood of the wall than in the boundary layer proper.

By averaging the instantaneous signals from the circular probes, time-averaged local mass transfer coefficients and Sherwood numbers were deduced. Typically, the averaging was extended over a 3 min interval which, in light of the preceding discussion, should be sufficient to provide meaningful results. To facilitate the examination of the Rayleigh number dependence and to achieve a compact presentation, the results are plotted with $Sh_x/Ra_{\theta,x}^{\frac{1}{3}}$ on the ordinate and $Ra_{\theta,x}$ on the abscissa. If the data were to follow the one-third power dependence, then they would fall on a horizontal line in such a graph.

A presentation of the time-averaged local mass transfer results is made in Fig. 4 for inclination angles of 0, 10, 15, 20, 30 and 45 deg. The results for each inclination angle are referred to a specific ordinate scale, either at the left or at the right as indicated by the arrows. The lowest Rayleigh number for each set of data is that which corresponds to the onset of instability. Aside from the vertical plate, for which the Rayleigh number range is too small to permit conclusions to be drawn, the results for the other cases clearly indicate an approach to a one-third power dependence at the higher Rayleigh numbers as characterized by the horizontal dashed lines. At inclination angles of 20, 30 and 45 deg, the pattern of the data at lower Rayleigh numbers is an initial rise and then a fall, and this is believed to correspond to the transition regime.

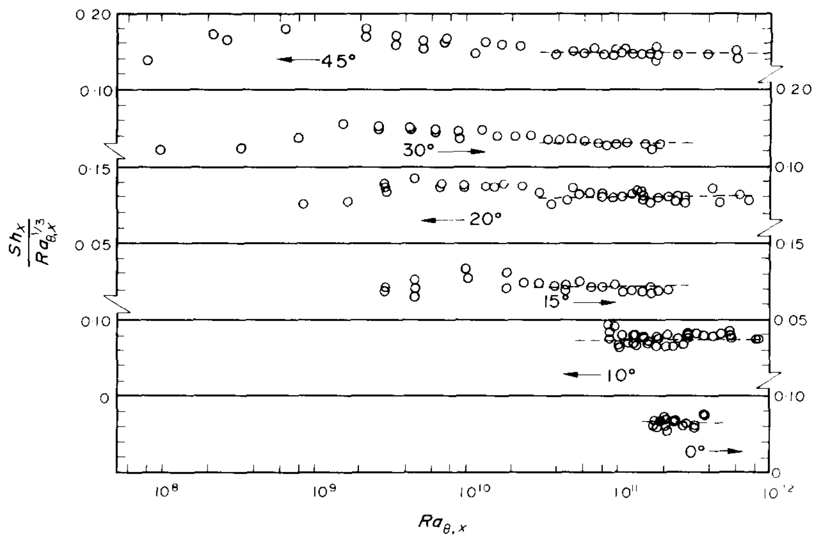

FIG. 4. Time-averaged local mass transfer results in the turbulent and transition regimes.

Further inspection of Fig. 4 indicates that $Sh_x/Ra_{\theta,x}^{\frac{1}{3}}$ varies appreciably with the inclination angle. Therefore, the use of $g\cos\theta$ in lieu of g is not sufficient to account for the influence of plate inclination in the transition and turbulent regimes. The effect of inclination angle in the turbulent regime may be characterized by a parameter $C(\theta)$ defined by

$$Sh_x = C(\theta)\, Ra_{\theta,x}^{\frac{1}{3}}. \qquad (15)$$

Numerical values of $C(\theta)$ were determined by using equation (15) in conjunction with the turbulent data appearing in Fig. 4, resulting in a fit given by the horizontal dashed lines. These C values are plotted in Fig. 5 as a function of the

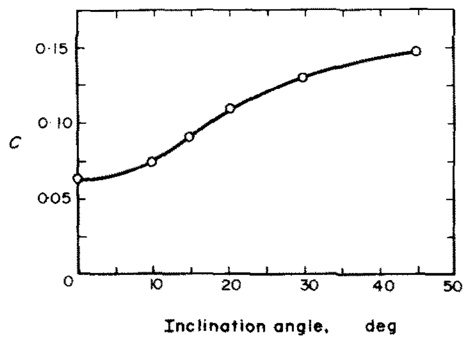

FIG. 5. Angle dependence of the coefficient $C = Sh_x/Ra_{\theta,x}^{\frac{1}{3}}$.

inclination angle. The figure shows that C varies appreciably over the range of angles investigated and that substantial errors can be made by neglecting this variation. It might also be noted that the C value for the vertical plate is tentative owing to the paucity of data.

The only prior work on turbulent, natural convection mass transfer appears to be that of Fouad and Ibl [14], who measured overall results for the vertical plate in the Rayleigh number range from 2×10^{11} to 10^{15}. Their overall mass transfer measurements included contributions from the laminar, transition, and turbulent regimes, thereby precluding a direct comparison with the local turbulent mass transfer results obtained here.

There is a substantial body of information on turbulent, natural convection heat transfer, most of which is concerned with the vertical plate. A study of the available literature reveals the absence of a consensus. For instance, the analyses of Eckert and Jackson [16] and of Bayley [17] respectively yield $\frac{2}{5}$ and $\frac{1}{3}$ power Grashof number dependences, whereas the more recent analysis of Kato and co-workers [18] gives a power ranging from 0·31 to 0·36, depending on the Prandtl number. Among recent experimental investigations, carried out either in air or water, the power of the Grashof number dependence was $\frac{1}{3}$, 0·365, 0·41, 0·28 and $\frac{1}{3}$, respectively reported in [19–23].* The experimental results of Fujii and co-workers [6], encompassing both water and oils, were correlated with either a $\frac{1}{3}$ power representation or a $\frac{2}{5}$ power representation, depending on the fluid. The just-mentioned disparities emphasize the difficulties inherent in performing natural convection heat transfer experiments and, in this way, call attention to the advantages of the electrochemical mass transfer method.

For inclined surfaces, turbulent heat transfer results are reported by Vliet [4] for the uniform heat flux boundary condition, with water as the primary working fluid (two runs were also made in air). A correlation involving the 0·32 power of the Grashof number was obtained.* Furthermore, the correlation, based on g rather than on $g \cos \theta$, is purported to be independent of the inclination angle. The data were plotted on conventional logarithmic scales with a high density of points, so that it is difficult to discern whether the scatter ($\sim$25 per cent) is related to variations of the inclination angle.

In any event, the effect of inclination on turbulent transfer appears to be substantially smaller in Vliet's work than in the present experiments. These different behaviors might be due, at least in part, to the different boundary conditions at the plate surface (uniform heat flux versus uniform concentration). Indeed, in connection with turbulent heat transfer studies for the vertical plate, Vliet and Liu [22] cautioned

* Results reported in [22] and [4] were based on a modified Grashof number appropriate to uniform wall heat flux. For present purposes, the correlations of [22] and [4] were recast in a form involving the conventional Grashof number.

against the expectation of similar behavior for the uniform heating and uniform temperature boundary conditions. Another factor which may be relevant is that the Schmidt number of the present experiments is about 500 times larger than the Prandtl number of Vliet's tests, thereby altering the relative sizes of the participating boundary layers.

Spanwise variations in the transition regime

The flow visualization studies reported in [1] indicated that for inclination angles greater than 17 deg, the breakdown of the laminar flow regime for thermal natural convection in water is characterized by longitudinal vortices distributed across the span of the plate. On the other hand, for angles less than 14 deg, the onset of transition was found to be characterized by plane waves which are more or less uniform across the span. The two modes of instability co-existed in the range between 14 and 17 deg. With the foregoing as background, attention is now turned to the measurements of spanwise mass transfer distributions.

As was described in the Apparatus section, an ensemble of 55 probes was fabricated to facilitate the spanwise measurements. However, during the course of the experiments, the insulation between the probes was attacked by the sulphuric acid in the electrochemical solution, thereby causing short circuits. As a consequence, the number of active probes decreased with time. Fortunately, a substantial amount of data was collected before a large portion of the ensemble became inactive.

The digital voltmeter employed to measure the instantaneous shunt currents was able to *sequentially* read 24 inputs, with a 0·3 s interval between each reading (i.e. all 24 probes were read in about 7 s). Typically, a set of 24 probes was scanned for a period of $1\frac{1}{2}$ min and then, following an interval of about 45 s required

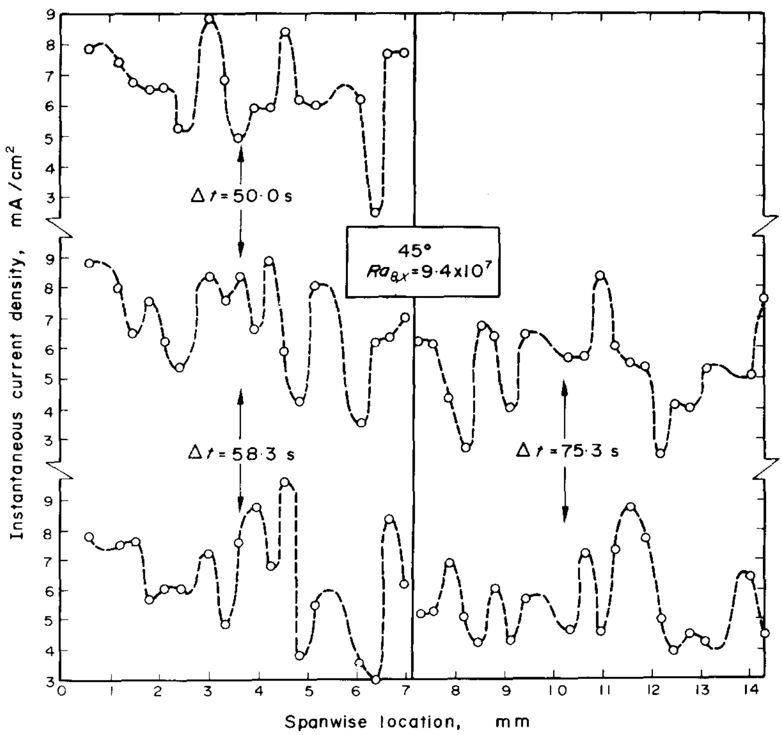

FIG. 6. Instantaneous spanwise mass transfer distributions in the transition region, inclination angle = 45 deg.

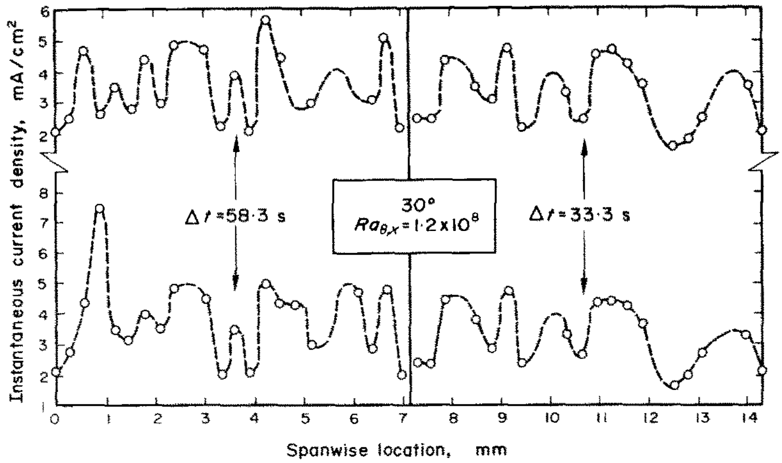

FIG. 7. Instantaneous spanwise mass transfer distributions in the transition region, inclination angle = 30 deg.

for changing connections, a second set of 24 probes was scanned for $1\frac{1}{2}$ min; and so forth. The data thus obtained were cross plotted to obtain spanwise distributions corresponding to a given instant of time.

Instantaneous spanwise mass transfer distributions for inclination angles of 45, 30, 20 and 15 deg are presented in Figs. 6–9 respectively. The ordinate variable is the current density, which is directly proportional to the mass transfer rate [equation (5)]. The abscissa variable is the spanwise location, measured relative to the first probe in the 55-probe ensemble. Spanwise averages of the data appearing in Figs. 6–9, when averaged over time, agree well with the values plotted in Fig. 4.

The Rayleigh number characterizing each figure was chosen to be indicative of the transition regime for that angle of inclination. The range of the transition regime was gauged from Figs. 2 and 4 and from the presence or absence of streaks on the plate surface indicating spanwise preferential deposition of copper ions.

Figures 6–9 have a common structure. In each figure, data from probes 1–24 are plotted in the left half, while data from probes 25–48

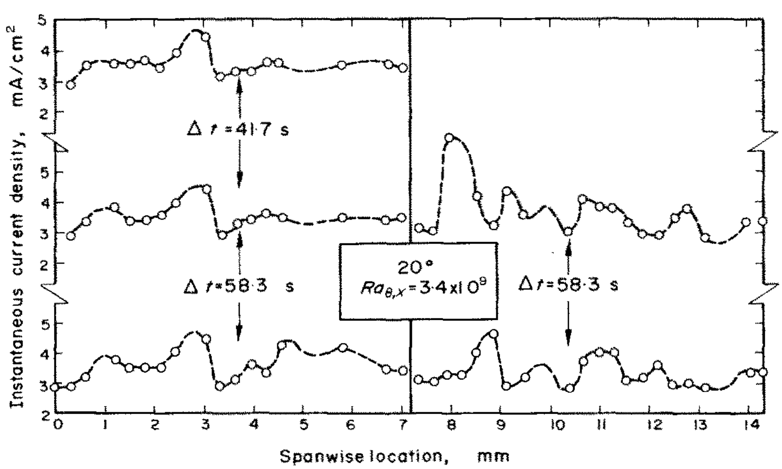

FIG. 8. Instantaneous spanwise mass transfer distributions in the transition region, inclination angle = 20 deg.

are shown in the right half. To provide a clearer picture, dashed curves have been faired to interconnect the points for a given instant of time. The time lapses between the successive instantaneous distributions are indicated on the figures, as are the Rayleigh numbers. Owing to the data acquisition procedure, adjacent distributions plotted in the left and right portions of the figures do not correspond to the same instant of time.

Turning first to Fig. 6 (inclination angle = 45 deg), it is seen that the instantaneous mass transfer varies in an oscillatory manner across the span, with the peak-to-valley variations being approximately equal to the spanwise average. The distance between adjacent peaks or valleys is about one mm. It is also seen that the spanwise distributions shift with time, indicating a corresponding shifting of the longitudinal vortices. However, as will be demonstrated shortly, there are preferred locations for the vortices.

As the angle of inclination decreases (Figs. 7–9), the relative amplitudes of the spanwise variations become smaller, and the shifting of the distributions with time diminishes. These trends are believed to hold although there is a possibility that the flow conditions of Figs. 6–9 correspond to earlier or later stages of transition. The spanwise distance between peaks and valleys continues to be about one mm. The results for the inclination angle of 15 deg are especially interesting. In this case, only an occasional peak or valley is visible, separated by regions of nearly uniform mass transfer. Thus, the present mass transfer measurements support the visual observations of [1] with respect to the disappearance of the longitudinal vortices at inclination angles less than about 15 deg.

There are clear indications that, although they shift with time, the longitudinal vortices have preferred locations. In this connection, time-averaged spanwise distributions are plotted in Fig. 10, each point being an average over $1\frac{1}{2}$ min. Results are given for inclination angles of 45, 30, 20 and 15 deg, for the Rayleigh numbers which are respectively indicated in Figs. 6–9. If the positions of the vortices had been random, then the time-averaged distributions would be uniform. The fact that there is a pattern of maxima and minima in the time-averaged distributions testifies to the existence of preferred locations. From a comparison among the relevant figures, it is seen that the amplitudes of the spanwise variations are somewhat smaller in the time-averaged distributions than in the instantaneous distributions.

Another indication of the existence of preferred locations was the presence of streaks on the plate surface, corresponding to a greater deposition of copper ions.

As a final experimental objective, it was planned to examine the restoration of spanwise

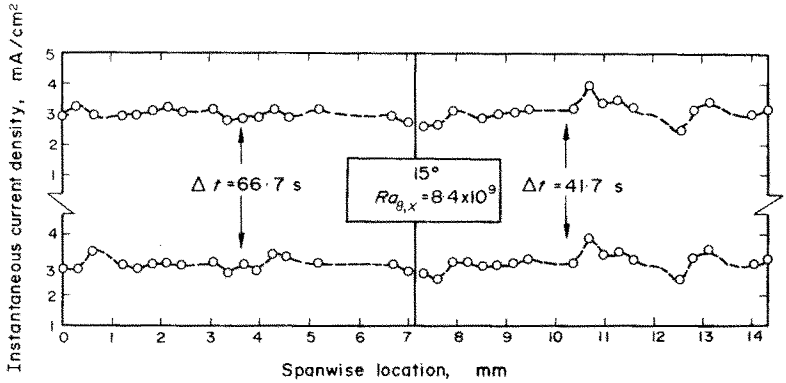

FIG. 9. Instantaneous spanwise mass transfer distributions in the transition region, inclination angle = 15 deg.

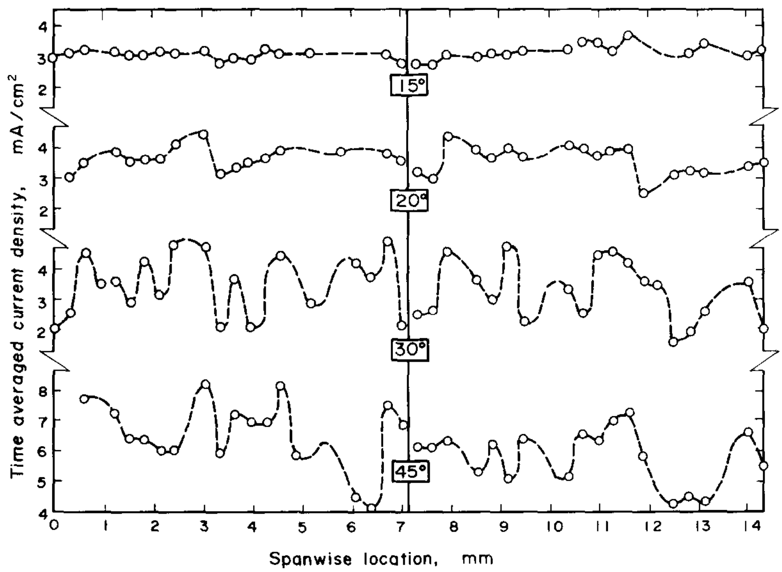

FIG. 10. Time-averaged spanwise mass transfer distributions in the transition regime.

uniformity as the Rayleigh number is increased and fully turbulent conditions are attained. Unfortunately, the 55 probe ensemble became inoperative before this objective was fully realized, but sufficient data were collected to establish a clear trend. Figure 11 shows instantaneous spanwise mass transfer distributions for two Rayleigh numbers at an inclination angle of 30 deg. It is seen that with increasing Rayleigh number, the relative amplitude of the spanwise variations decreases markedly, pointing to an eventual restoration of spanwise uniformity.

CONCLUDING REMARKS

Electrochemical mass transfer measurements have been shown to be a highly effective means for obtaining local information on both the streamwise and spanwise distributions of natural convection transfer coefficients. In addition, since the mass transfer probes are inertia free,

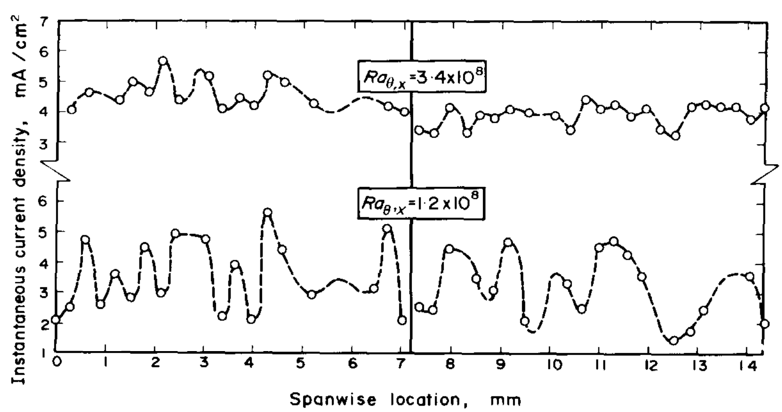

FIG. 11. Effect of Rayleigh number on the instantaneous spanwise mass transfer distribution, inclination angle = 30 deg.

timewise variations can be measured. Other advantages of the electrochemical method are (a) well defined boundary conditions, (b) small fluid property variations, (c) high precision of measurement, and (d) small ambient disturbances.

Excellent agreement was found to exist between laminar theory and the experimentally determined local mass transfer coefficients, both for vertical and inclined surfaces. The onset of transition occurred at a Rayleigh number slightly greater than 10^{11} for the vertical plate and, with increasing surface inclination, the instability Rayleigh number decreased markedly. In the transition and turbulent regimes, the local mass transfer rates varied with time. Both the period and the amplitude of the timewise turbulent fluctuations were appreciably larger for inclined surfaces than for the vertical plate. The time-averaged turbulent mass transfer coefficients exhibited a one-third power dependence on the Rayleigh number and, in addition, a strong dependence on inclination angle. In the transition regime, for surfaces inclined at angles greater than 15 deg, significant spanwise variations in both the instantaneous and time-averaged mass transfer rates were measured.

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CONVECTION NATURELLE LAMINAIRE OU DE TRANSITION OU TURBULENTE ADJACENTE À DES SURFACES INCLINÉES ET VERTICALES

Résumé—On a étudié expérimentalement à l'aide d'une technique électrochimique le transfert massique en convection naturelle adjacente à des plaques verticales et à des plaques inclinées à face active tournée vers le haut. Des mesures locales sont faites pour déterminer les distributions de transfert massique à la fois dans l'écoulement et sur la surface. Des flux massiques dépendants du temps ont été mesurés dans les régimes de transition ou turbulents.

Les coefficients locaux de transfert massique laminaire s'accordent très bien avec les estimations théoriques pour des surfaces verticales ou inclinées. Le nombre de Rayleigh marquant la transition laminaire-turbulent varie nettement avec l'angle d'inclinaison et décroit quand on augmente l'écart à la position verticale. Dans le régime de transition, pour des surfaces inclinées avec des angles supérieurs à 15° par rapport à la verticale, on a mis en évidence des nettes variations le long de la surface pour les flux massiques instantanés comme pour les flux moyens. Les fluctuations en fonction du temps des flux massiques turbulents sont d'amplitude et de période nettement plus grandes pour des surfaces inclinées que pour la plaque verticale. La valeur moyenne des coefficients locaux de transfert massique turbulent est reliée à la puissance $\frac{1}{3}$ du nombre de Rayleigh et elle dépend fortement de l'angle d'inclinaison.

FREIE KONVEKTION AN GENEIGTEN UND SENKRECHTEN OBERFLÄCHEN BEI LAMINARER, TURBULENTER UND GEMISCHTER STRÖMUNG

Zusammenfassung—Es wurden Versuche über den Stoffübergang bei natürlicher Konvektion längs senkrechter und geneigter, nach oben zeigender Flächen durchgeführt unter Verwendung eines elektrochemischen Verfahrens. Örtliche Messungen lieferten Stoffübergangsverteilungen in und quer zur Strömungsrichtung. Die Zeitabhängigkeit des Stoffübergangs wurde im turbulenten und im Übergangsgebiet gemessen.

Die gemessenen Stoffübergangskoeffizienten stimmten sowohl für senkrechte als auch für geneigte Flächen sehr gut mit analytischen Voraussagen überein. Die Rayleigh-Zahl, die den Umschlagpunkt laminar–turbulent angibt, änderte sich spürbar mit dem Neigungswinkel in der Weise, dass sie mit zunehmender Abweichung von der Senkrechten abnahm. Im Übergangsgebiet zeigten sich für Flächen mit Neigungswinkeln grösser als 15° zur Senkrechten deutliche Schwankungen sowohl der momentanen als auch der zeitlich gemittelten Stoffübergangsdichte senkrecht zur Strömungsrichtung. Die zeitlichen Fluktuationen der turbulenten Stoffübergangsdichte hatten an geneigten Flächen eine wesentlich grössere Amplitude und Periode als an der senkrechten Platte. Die zeitlich gemittelten, örtlichen turbulenten Stoffübergangskoeffizienten wurden mit der dritten Wurzel der Rayleigh-Zahl korreliert. Ausserdem weisen sie eine merkliche Abhängigkeit vom Neigungswinkel auf.

ЛАМИНАРНАЯ, ПЕРЕХОДНАЯ И ТУРБУЛЕНТНАЯ ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ ВБЛИЗИ НАКЛОННЫХ И ВЕРТИКАЛЬНЫХ ПОВЕРХНОСТЕЙ

Аннотация—Эксперименты по переносу массы при естественной конвекции вблизи вертикальных и наклонных пластин выполнялись с помощью электрохимической методики. Были проведены локальные измерения, в результате которых получены распределения переноса массы по потоку и в пространстве. Измерялись зависящие от времени скорости переноса массы в переходных и турбулентных режимах.

Измеренные локальные ламинарные коэффициенты переноса массы хорошо согласовывались с аналитическими расчетами как для вертикальных, так и для наклонных поверхностей. Число Релея, при котором возникает переход от ламинмрного режима к турбулентному, подвергалоср значительным изменениям с изменением угла наклона, уменьшаясь с увеличением отклонения от вертикального положения. В переходном режиме для поверхностей, наклоненных под углом более 15° к вертикали, очевидны, значительные пространственные изменения как в мгновенных, так и в усредненных по времени скоростях переноса массы.

Временные колебания скоростей турбулентного переноса массы имели значительно большую амплитуду и переход для наклонных поверхностей нежели для вертикальной пластины.

Усредненные во времени локальные коэффициенты турбулентного переноса массы коррелировались с числом Релея в степени 1/3 и, кроме того, показали значительную зависимость от угла наклона.